Evaluating Engagement Level and Analytical Support of Interactive Visualizations in Virtual Reality Environments

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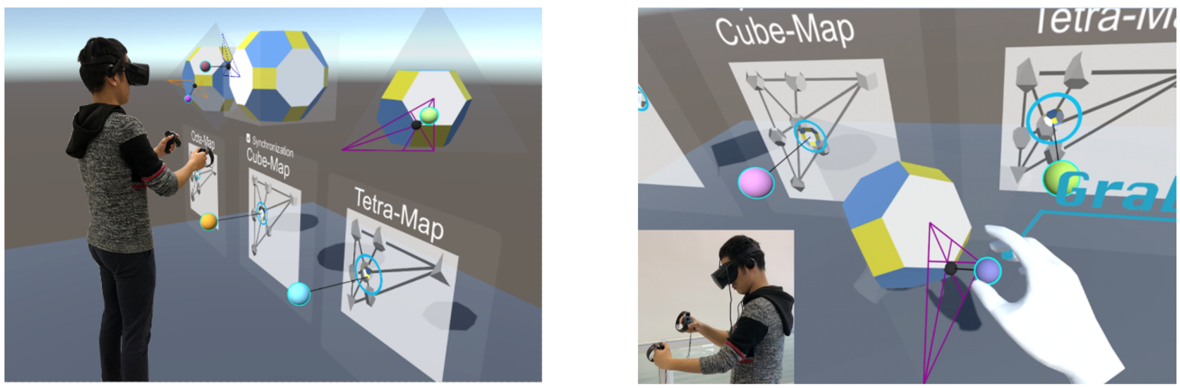

Figure 1: Two views of a user interacting with the visualization tool for virtual reality environments that allows users explore the transformational and structural properties of 3D shapes. The tool contains a number of interactive visualizations that are linked together and is used to evaluate the level of engagement and analytical support of visualizations in virtual reality systems.

Abstract

Interactive visualizations are external cognitive artifacts aimed at supporting users' exploratory and sense-making activities. In recent years, there has been an explosion of commercial virtual reality (VR) head-mounted displays (HMD). These VR devices are meant to offer high levels of engagement and improve users' analytical exploration of the displayed content. However, given their rapid market introduction, the possible influences and usefulness that VR could bring in terms of supporting users' exploration with interactive visualizations remain largely underexplored. We attempt to fill this gap and provide results of an empirical study of an interactive visualization tool that we have developed for a VR HMD system. This tool is aimed at facilitating exploratory and analytical reasoning activities with 3D shapes and their transformational processes. Overall, the results show that the tool is supportive of users' exploratory and analytical activities based on the significant improvement in their post-experiment test scores (when compared to their pre-experiment ones) and their engagement level measured via a user engagement questionnaire and participants' comments. The results shed a positive light on the use of visualizations in VR environments and can inform the design of these tools of domains beyond 3D transformational geometry.

Index Terms: H.5.1 [Information Interfaces and Presentation]: Multimedia Information Systems—Artificial, augmented, and virtual realities; H.5.2 [Information Interfaces and Presentation]: User Interfaces—Interaction Styles

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1 Introduction

Interactive visualization tools are primarily aimed to engage users in exploratory, analytical, and sense-making activities [4, 6, 25, 34]. While visualizations have been used widely in desktops and mobile devices, it is more recent that they are included in virtual reality (VR) environments. VR technologies have become one of the most popular and exciting emergent platforms that can transform how we interact with visually-enhanced objects. The coupling between visualizations and VR can bring further benefits to user-visualization exploration, including a greater level of engagement due to the immersive nature of VR. However, with the rapid introduction of VR, the possible effectiveness and usefulness of the interactive visualization tools built on this technology still remain largely underexplored.

To fill this gap, the purpose of this research is to examine whether immersive VR interactive visualization tools can support and engage users in analytical reasoning, exploratory sense-making, and knowledge acquisition. To this end, we designed and developed Virtual Reality Solid Visualization Tool (VRSVT), a mathematical visualization tool for head-mounted display (HMD) systems, with five individual visuo-interactive techniques (Solid Transition Maps, Synchronization, Dot-Manipulation, Rotation, and Geometric Magnification), and then conducted a user study to assess its support of users' analytical, sense-making, and exploratory activities. VRSVT, which will be described in more detail later, aims to support the exploration of 3D geometrical shapes regarding their individual structural properties as well as their transformative processes through which one shape can be derived from another. In our study, pre- and post-test scores are used to measure if the tool could increase users' understanding of the mathematical concepts. In addition, a user engagement questionnaire [20] is employed to assess whether the tool would be able to achieve an adequate level of engagement from users. Moreover, we also investigated users' affective responses towards VRSVT and towards each individual visualization technique contained in the tool.

In this paper, we report the results of our study in terms of the participants' knowledge improvement, engagement level, and affective response to VRSVT. Overall, the average performance of participants has shown significant improvements in their knowledge of the subject after interacting with the tool. They have also shown to have a high engagement level with this type of immersive visualizations.

Before reporting the results of the study, in the next section we present some previous work related to our research. In the third and fourth sections, we delineate the research methodology of our study, including the design rationale of VRSVT and the experiment design framework. In the section after, we describe the result of the study in detail. Finally, in the last two sections, we discuss the findings, limitations and future work, and summarize the conclusions of this work at the end.

2 Related Work

In this section, we introduce some previous work with respect to VR and interactive visualization tools, tools for exploring 3D geometry, which is the domain of this work, and assessing engagement levels in user-visualization interaction.

2.1 VR and Interactive Visualization Tools

Interactive visualization tools are external artifacts intended to support and enhance users' exploratory, interpretational and sense-making processes involving visually-represented information [4, 6, 25, 34]. These external aids help to increase memory, reasoning, and make users 'smarter' [40]. When turning these tools into VR environments, computer-generated 3D visualizations are provided to surround the users and enable them to see the representations from different angles and interact with 3D objects in a way that is closer to how they would normally interact with physical objects [8, 46]. Such VR tools usually allow users to 'reach' into the visualizations and manipulate them to assist their reasoning and analytical processes. This includes exploring new findings, formulating and testing hypotheses, as well as interpreting and reaching their own conclusions. Compared to normal visualizations displayed in 2D screens, VR tools provide users with deeper levels of interaction possibilities, and are claimed to decrease social anxiety, improve motivation, and enhance engagement [3, 15, 38, 43, 63]. Because of these affordances, the use of VR interactive visualization tools has become increasingly promising, especially following the recent release of many commercial VR systems like the Oculus RIFT and HTC Vive. There has been some work applying this emergent technology to support the exploration and sense-making of abstract visual concepts [18, 39, 49, 57]. However, empirical evaluations with users in terms of performance improvement and their engagement levels with these tools are still an area that can benefit from further investigations — this is one key motivation for our current work.

2.2 3D Geometry

3D geometric solids are chosen as the test-bed domain for our experiment. Within 3D geometry, we focus on Platonic and Archimedean solids. Platonic solids are 3D shapes that are composed of only one type of regular polygons — that is, a polygon with all its edges of equal length [64] (see Fig. 2). Archimedean solids are defined as 3D shapes composed of two or more types of regular polygons. Platonic and Archimedean solids are closely interconnected and can

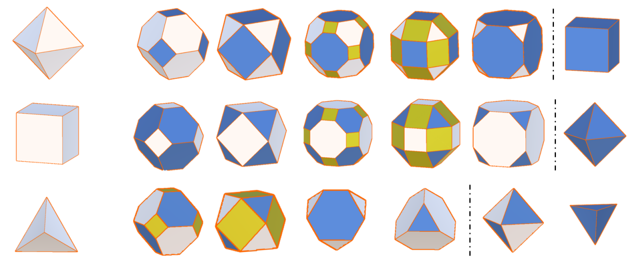

Figure 2: (Top to bottom rows) Octahedron, Cube (or Hexahedron) and Tetrahedron (LEFT) and all their derivable Archimedean solids (Note: the octahedron, which can also be derived from the tetrahedron, is not an Archimedean solid) (RIGHT).

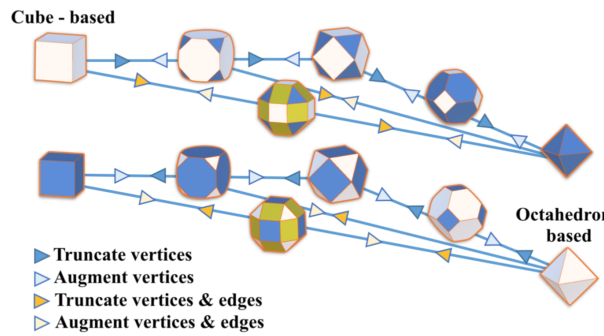

Figure 3: Multi-layered relationships between Platonic and Archimedean solids and the processes of morphing from one to another.

be obtained from one another. Fig. 2 shows all Archimedean solids that can be derived from the octahedron, cube, and tetrahedron (i.e., the three Platonic solids).

Because of their internal properties and transformational possibilities, polyhedrons are also difficult to explore, analyze, and learn without the aid of external tools; they might even sometimes make people frustrated when they have to study and reason with them [44]. To understand these characteristics, one is required to think and imagine how the shapes can morph and transform themselves from one to another. Additionally, the relationships between Platonic and Archimedean solids are multi-layered. These relationships not only deal with planes of symmetry, but also are built upon how the shapes can be obtained from each other by truncating/augmenting their vertices and edges (See Fig. 3) [34].

From Figure 3, one can observe that the shapes contain three types of spatial knowledge: (1) *landmark*, which is the position of the regular solids in the transformation processes; (2) *route*, indicated by the transitional/navigational routes that allow the solids to morph from one and become another solid; and (3) *survey*, the entire landscape of landmark (regular) solids and morphing routes. Such rich spatial attributes make the set of shapes a feasible testbed for evaluating how a VR-based visualization tool about these shapes can support sense-making, analytical reasoning, and knowledge acquisition.

In this research, we are concerned with the exploration and learning of 3D geometry, and the use of a VR-based interactive visualization tool to investigate whether it is supportive of these exploratory and sense-making activities. In particular, we are focusing on two aspects of sense-making activities: (1) knowledge acquisition from the visualizations, and (2) user engagement with these types of external cognitive aids.

2.3 Visualization Tools for the Exploration of 3D Geometry Concepts

The ability to visualize abstract geometric structures and their spatial relationships can help learners with their mathematical thinking and learning. Some learners, because of the difficulty of mentally manipulating 3D shapes, find them challenging to explore [34]. To help the users with sense-making of abstract concepts, external visualization learning tools have been extensively studied in desktop PCs [13, 19, 36, 52, 54] and mobile devices [5, 22, 45, 55]. This prior research has indicated that interactive visualization tools can help students improve their understanding of geometric concepts [1, 33, 35]. However, as some other studies demonstrate, there may not be any significant improvements in students' understanding of 3D geometry concepts from 2D interfaces [1]. In another research, Liang and Sedig [34] found a significant improvement of students after they interacted with a visualization tool of 3D shapes but are displayed on a 2D desktop screen. One reason for this significant improvement might because their study involved long and regular daily training time (three sessions of 45 minutes on three consecutive days) for students to interact with their tool.

In recent years, virtual reality technologies have developed rapidly and have the potential to improve the visualization of and interaction with geometric shapes, especially those of three or higher dimensions. Given their inherent 3D nature, virtual reality systems could be better suited for exploring these concepts. This is because they allow users to look at the visualizations from different angles, replicating as close as possible the way how one looks at them as actual physical objects. Despite their benefits, research describing currently marketed VR HMD, such as the Oculus RIFT, for visualizing geometric objects in 3D environments is only emergent and underexplored. Kaufmann and his colleagues have developed Construct3D, a 3D geometric construction tool specifically designed for mathematics and geometry education, and made a series of analysis with this tool [26–29] but did not provide empirical data about participants' performance before and after using their tool. Lai et al. have introduced Geometry Explorer [32], a VR based tool that allows users to view and manipulate the dimensions of 3D shapes but likewise have not reported any formal empirical studies looking at performance improvements and the engagement levels.

Our research aims to fill this gap by systematically studying if VR visualization tools can lead to performance improvements, high levels of engagements, and positive user subjective feedback.

2.4 Assessing user's engagement with visualizations

Prior research has found a close link between students' engagement with their persistence, academic achievement, and satisfaction with learning [7, 12, 21, 24, 50, 66]. In the same way, user engagement with visualizations is an important aspect of the exploratory process and, as such, it can be used to gauge the effectiveness of the tool. According to Henrie et al. [17], there are mainly three types of measurements in the literature used for gauging engagement: (1) quantitative self-reporting, (2) qualitative measurements, and (3) quantitative observational approaches. Quantitative self-reporting measures the engagement of people through surveys and questionnaires by asking users to report their perceived level of engagement with a tool [14, 20]. This type of techniques is frequently used because of its usefulness for investigating unobservable aspects of user engagement such as emotional and affective experiences [7, 56]. However, variance in people's engagement across time is difficult to capture using this technique alone [17]. An alternative approach is to use qualitative measurements which include direct video, or screen captured observations of users' behavior while performing learning or other sensing making activities [11], and also interviews, focus groups, or analysis of discussions [16, 37]. These approaches are useful for exploratory studies but are also quite challenging to scale and quantify the engagement level. Some other researchers also

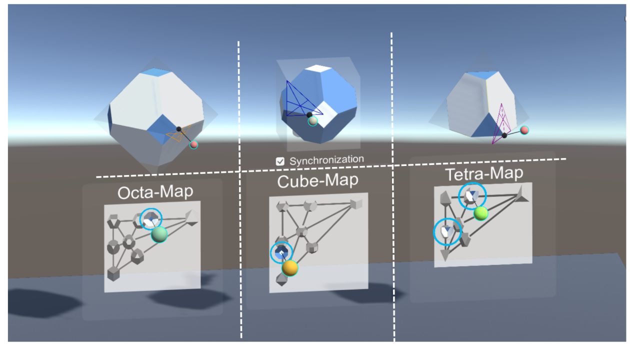

Figure 4: The main interface of VRSVT. It is divided implicitly into two sections: (TOP) the base solids enlarged for close inspection; (BOTTOM) the Solid Transition Maps, each corresponding to one solid showing how other shapes can be derived from the base, Platonic solid.

used quantitative measurements to quantify people's behaviors, such as frequency of clicking, in order to determine the level of engagement during knowledge acquisition [2, 9, 16]. However, only limited aspects of engagement can be obtained through this approach.

In this study, we employed two of the above-mentioned methods, quantitative self-reporting and qualitative measures, to assess users' engagement level. We used the 7-point Likert Scale user engagement questionnaire proposed by Hung and Parsons [20], called VisEngage, to assess users' engagement level when using VRSVT. VisEngage quantifies 11 engagement characteristics with 22 seven-point Likert-scale questions. We used this self-assessment questionnaire because it is one of the more complete and comprehensive questionnaires for evaluating user engagement with visualizations. It includes a number of important aspects of user experience related to how users can engage with a visualization tool. Moreover, we used videos to record our participants' behavior when interacting with VRSVT and post-experiment interviews with participants to capture their subjective feedback. These data would help us assess how engaged the participants were when interacting with our visualization tool.

3 Description and Design Rationale for VRSVT

In the VR visualization tool we developed, users are able to manipulate and interact with the 3D shapes that can be derived from the three Platonic solids: Cube, Octahedron, and Tetrahedron. Various visuo-interactive features are designed and included in VRSVT to provide users with a variety of complementary techniques that are aimed at supporting their exploratory needs and sense-making activities.

3.1 Apparatus

The experiment was conducted on a PC with an i7 CPU running at 4 GHz and equipped with a GTX1080 Ti GPU. We used the Oculus Rift CV1, a popular commercial VR device, in the experiment. It has a 1080*1200 display resolution for each eye and comes with its customized dual-hand controller, the Oculus Touch, which was used to allow participants to interact with the different components of the tool. The Oculus Touch provides intuitive feedback and comes with a rich set of gestures.

3.2 Interface

Fig. 4 shows the main interface of VRSVT. The interface consists of two major areas: The Main Shapes and Solid Transition Maps (STM), which are described in later sections.

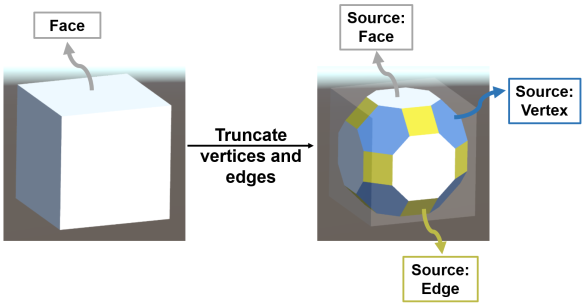

Figure 5: (LEFT) The cube and (RIGHT) the rhombi-truncated cuboctahedron with three different colors indicating the transformational process (white faces ↔ original sides; yellow faces ↔ truncated edges; blue faces ↔ truncated vertices).

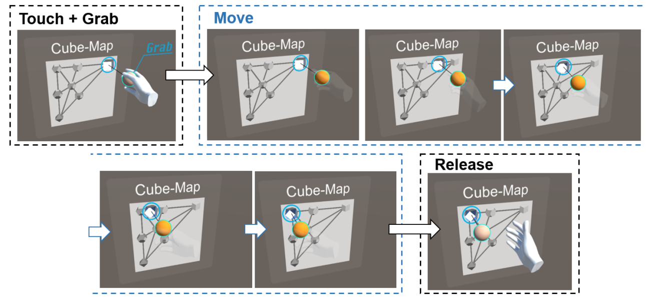

Figure 6: Screenshots showing the three stages of interacting with an STM: Grab-Move-Release.

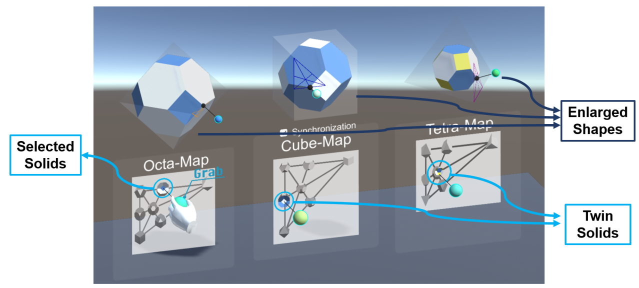

Figure 7: Screenshots showing how the Synchronization feature works. It allows all three STM and Enlarged Solids to be dynamically linked. It is especially useful for exploring the relationship between the Platonic and Archimedean solids.

3.3 3D Solid Visualizations

The 3D solid visualizations are placed in the middle of the interface, which serves as the most important part of the visualization tool. They present explicit structural information about the solid being observed. To provide further transformational information, the faces of solids are rendered with different colors. According to our review, color has been shown to play an important role in visual communication, which can significantly improve user cognition at the conceptual and subconscious level [8, 46]. In our tool, three colors (i.e., white, blue, and yellow) are used to render a solid's faces, with each color indicating the source of origin. Fig. 5 shows a rhombitruncated cuboctahedron. The blue and yellow faces indicate the result of truncating both vertices and edges respectively.

3.4 Visuo-Interactive Techniques

Five visuo-interactive features were integrated into the tool, which together were aimed at facilitating the exploration and analytical reasoning with the solids, especially their relationships with each other. We describe each feature in the next few subsections (please also refer to the supplementary video which shows how they behave in real time).

(1) *Solid Transition Maps*. Solid Transition Maps (STM) were designed to support active navigation within and between visualizations because this activity is considered an important form of exploratory interaction [34, 53]. Thumbnails of shapes are connected by lines to indicate the transitional processes of how shapes can be derived from each other; as such, the lines are in a way navigation paths and let users form a cognitive and conceptual map of all related solids [23, 47, 61]. For instance, in the second row of Fig. 2, the STM of the Cube consists of the cube itself (the leftmost shape) and regular solids derived from the cube (the six solids on the right). By following the top horizontal navigational path from the cube, one can derive first the *'truncated cube'* and then the *'cuboctahedron'* after (See Fig. 6). In this sense, the three maps are intended, first, to support structural, transitional, and navigational reasoning within a given map, and, second, to facilitate comparative reasoning across the maps (See Fig. 4). The maps in VRSVT are interactive, each resembling a virtual panel. The reason why we use the panel interface is because a control panel is a common interface for interacting with devices in the real world. Also, panels are widely used in modern interactive computer applications. We thus leverage users' general familiarity with panels to facilitate the interaction process in the virtual reality environment [60]. Fig. 6 demonstrates the process of truncating all vertices of a Cube using STM. Users can manipulate and interact with each map using an interactive ball, by which the highlighted position on the map it points to is regarded as the current input.

(2) *Synchronization*. Apart from being able to display the currently selected solid (the one being interacted with by the user on the map), each STM is also capable of indicating the existence of *Twin solids* (See Fig. 7). A *Twin solid* is a shape that shares exactly the same structural properties with another shape but is derived from different transformational processes. Every possible shape that is derivable from the three main base solids (i.e., the cube, octahedron, and tetrahedron) has at least one Twin solid, which means that Twin solids always exist during the whole interaction process. The *Synchronization* function is designed to help visualize the existence and patterns of twin solids — This visual feature is used to indicate structural similarities across all three STM. It also ensures that the current Twin solid will always be displayed and moved in synchronicity during the exploratory process; changes occurring in one Twin solid will cause simultaneous changes in the three STM, if there is a connection.

(3) *Dot-Manipulation*. A local STM (LSTM) is placed on each map with an interactive dot located inside the map to semantically represent the location of in-focus selected solid (see Fig. 8a). This semantic magnification approach is inspired by semantic zooming, a visualization technique where the representation of an object is not only magnified but changes and additional details, often hidden from explicit, direct view, are unraveled and shown [42, 58]. The semantic dot is also interactive and behaves in a similar way as the interactive ball of each STM. Fig. 8b shows a sequence of the process of interacting with the semantic dot. The LSTM is located on a face of the cube and the (black) dot is on one of the vertices. The position of the dot indicates that the inner solid is the same as the base solid. Users can grab the interactive ball connected to the dot and move inside the LSTM to observe the transformation that takes place. To reinforce the connection between the STM and LSTM, the dot is synchronized with the STM, and this means that if the dot is moved, the corresponding interactive ball on the STM and the Selected/Twin solids will also change simultaneously. This

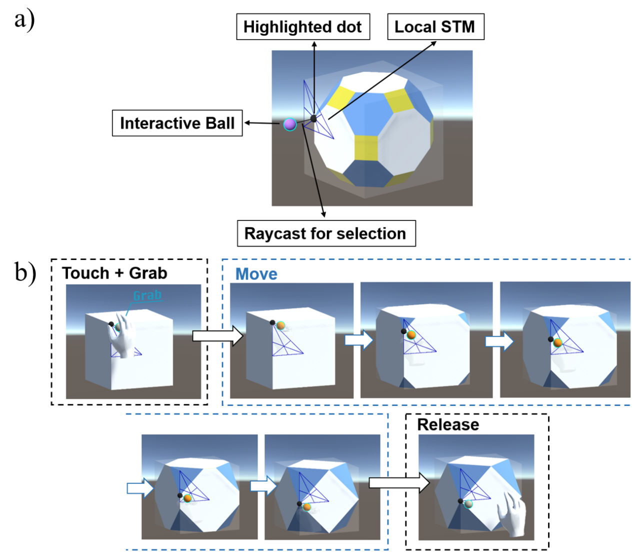

Figure 8: a) The Cube Enlarged Solid with its local Solid Transition Map (LSTM) mounted on one face of the wireframe outline; b) The three stages (Grab-Move-Release) of using dot-manipulation to truncate all the vertices of a cube.

leads to a consistent and flexible interaction, one in which cause and effect of an action can be continuously observed, thereby allowing users to observe a coherent, unified transitional process across all dynamically linked visualizations. These effects reinforce each other to enhance exploratory and sense-making analytical activities.

(4-5) *Rotation and Geometric Magnification*. These two interactions are based on one principle: Grab and manipulate. The entire Enlarged Solid can be grabbed and once grabbed it can be rotated in all directions (see Fig. 9a-b). That is, the users can reach out with their virtual hands and hold the solid so that it can be turned and viewed from different perspectives. This feature is particularly useful in 3D visualizations because of the occlusion effect which might lead to making it difficult to explore and analyze their structural composition that is hidden from direct view [10, 59]. In other words, this direct manipulation and rotation of the shapes allow users to view the shapes from different perspectives, thus mitigating the problem of occlusion.

The second interaction allows users to bring the shape closer or push it away. It is referred to as geometric magnification, a visualization technique by which the appearance of the object does not change but is simply made bigger, to allow for close examination, or smaller, to save space and deemphasize its importance among other competing visualizations (see Fig. 9c-d) [4, 58]. These two interactions of rotation and geometric zooming are integrated together for two reasons. For one, they complement each other — allowing users to look at the Enlarged solid closer and also from different viewing perspectives. The second reason is that together they can simulate better how a physical object can be explored in real life — by grabbing and rotating it to explore it from different distances and angles.

4 Usability Evaluation

In this section, we present the details of our evaluation of the tool.

4.1 Design

A multi-method (quantitative and qualitative) research design was used, including a number of data collection instruments, such as pre- and post-test, questionnaires and interviews. By applying this multi-method approach, we would be able to triangulate and cross-validate different observations. The comparison of pre- and post-test results would reveal performance improvements that VRSVT could lead to and why. Because participants only interacted with VRSVT between pre- and post-test, this interaction should be the cause of the performance differences between two tests. Moreover, the questionnaire and interviews would provide details about users' subjective preferences of the visuo-interactive features of the tool and its general level of usability and acceptance.

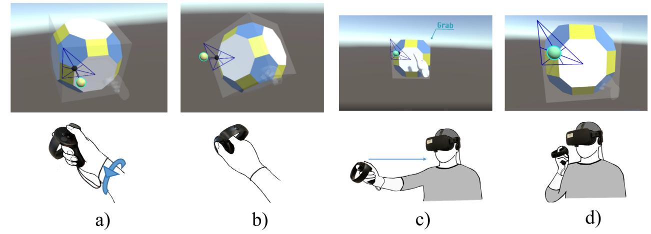

Figure 9: a-b): Rotating the wrist/controller to observe the shape from different perspectives; c-d): Bringing the hand/controller closer for a closer inspection of the visualization.

4.2 Subjects

Twenty undergraduate students (10 males and 10 females) aged between 18 and 25 ($M = 20.60$, $SD = 2.113$) years old were recruited to participate in the study. They were volunteers from a local university and came from different backgrounds. None of the students had used the tool before the experiment. All volunteers had normal or normal to corrected vision.

4.3 Procedure

The experiment was divided into five phases: (1) Understanding the content and procedures of our study; (2) Pre-test; (3) Performing pre-defined tasks with the tool; (4) Completing the VisEngage Questionnaire and a short interview; (5) Providing their subjective response to individual techniques, and (6) Post-test. The whole experiment took about 2 hours, which was similar to a typical weekly lab session they would attend. We also used video and screen captures of participants' interactions with the tool.

At the beginning of the experiment, a brief description of the procedure, content and some mathematical background of our study were introduced to all subjects; this took about 5 minutes. In the second phase, participants were asked to complete the pre-test. After, they were given time to become familiar with the tool, with a focus on understanding the five visuo-interactive features (i.e., solid transition maps, dot-manipulation, geometric magnification/zooming, rotation, and synchronization). This generally took about 15-20 minutes. In the third phase, they were asked to finish a set of question-based tasks while using our tool. The tasks were intended to provide participants with predetermined goals to facilitate data gathering within the short duration of the study. In addition, they would allow assess if the tool is supportive of users' knowledge acquisition within a short timeframe (as opposed to long training time in similar studies with non-VR systems, e.g. see [34]).

The tasks can be divided into three groups based on three types of spatial knowledge: Landmark, Route, and Survey (as discussed in the Related Work section). The tasks required participants to provide answers to them. Given that they would be wearing HMD goggles, it was not practical for the participants to use paper and pen. Instead, we developed a functionality to allow them to make annotations using the Oculus Touch controllers (see Fig. 10). Their answers were recorded automatically for later assessment. This part took about 40-50 minutes. In the fourth phase, subjects were asked to complete

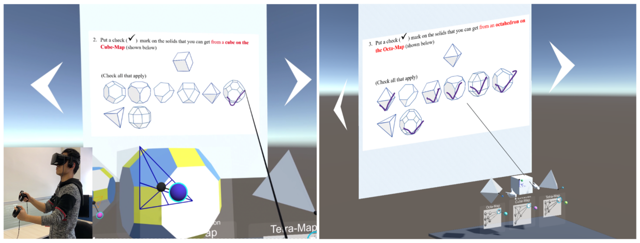

Figure 10: A sample task question in VRSVT anfd the annotation technique based on the ray-cast technique to allow participants to make markings.

the VisEngage questionnaire[1] [20] to measure their perceived level of engagement with the tool. The questionnaire consisted of twenty-two 7-point scale Likert questions with regard to 11 different types of user engagement measurements with visualizations. Afterwards, we interviewed participants about their feelings and perceptions of the tool and its supporting functions and if they saw any advantages and drawbacks. Participants were also asked to provide their affective response on a 5-point Likert-scale questionnaire. This part took about 30 minutes. Finally, in the last phase, participants were asked to complete the post-test, which contained the same questions as the pre-test. This took about 20 minutes. The language used in the test questions and tasks was plain and easy to understand. If participants were unclear about anything, they were encouraged to ask the researchers for clarification at any point during the study.

4.4 Data for Evaluating VRSVT

There are six sources of data that we collected to evaluate the VRSVT: (1) answers on the pre- and post-tests; (2) data from the VisEngage questionnaire; (3) data from Likert-scale questionnaire; (4) video and screen captures of participants' interactions with VRSVT, which contained verbal comments and body language; (5) transcripts of interviews; and (6) direct observations by researchers regarding how participants reacted and behaved during the experiment and their general usage together with body movement patterns.

5 Results

We next report our experimental results, which can be divided into two subsections: (1) Test achievement results, including a detailed statistical analysis of the data; and (2) Subjective feedback, which includes results from the engagement questionnaire plus participants' comments and feelings about the tool. We also present some observed behavioral patterns in this subsection.

5.1 Test Achievement Results

Fig. 11 (LEFT) shows a boxplot of the pre- and post-test scores. Participants generally achieved higher overall performance in the post-test.

Table 1 presents the descriptive data of the pre-test/post-test and the difference between the two for all participants to show the overall performance improvement. As can be observed, VRSVT led to a mean test score improvement of 20.00%, with the highest improvement of 44.44% (the lowest was 0%). The relatively large standard deviation on the test scores reveals that participants' level of understanding varied from before and after using the tool. Furthermore, the level of improvement also varied considerably among participants with a standard deviation of 12.67%. The individual performance data provide some clues to explain this variance. Some participants did not perform well during the pre-test but made a substantial increase on the post-test; for example from 11.1% to 55.5%. Other participants achieved relatively high scores on the pre-test, and thus had less space for a large increase. We also found one abnormal decrease; one participant had a high pre-test score of 50% but then it dropped to 33.3% in the post-test after using the tool. Based on our observations, this was due to the participant's unfamiliarity to VR devices, or sensibility to motion sickness caused by the participant's interaction with the VR system.

[1] VisEngage Sample Questionnaire: https://yahsin.github.io/VisEngage/

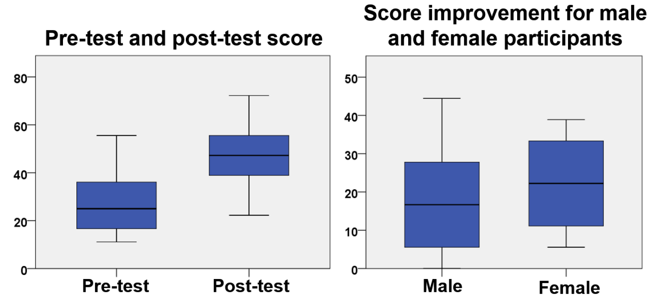

Figure 11: Boxplot for the overall pre- and post-test results in % (LEFT) and boxplot of score differences between pre- and post-test for male and female participants (RIGHT).

Table 1: Overall descriptive data for score in %.

	Mean	N	Std. deviation	Std. error mean
Pretest	27.222	20	12.860	2.875
Posttest	47.222	20	12.291	2.748
Improvement	20.000	20	12.669	2.832

A one-sided paired-sample t-test was performed to check whether there were significant changes between pre- and post-test scores. The result shows that participants performed significantly better (t_{diff} (19) = -7.011, $p < .001$). As such, participants' overall performance improved significantly after interacting with the tool. Because some prior research had shown that females and males could have different reactions towards VR (e.g., female users could be more susceptible to motion sickness [30, 41]), we also investigated whether gender differences in achievement scores existed. Fig. 11 (RIGHT) presents side-by-side boxplot results to compare the pre- and post-test improvements for both male and female groups. VRSVT generally resulted in homogeneous increment on the test scores, and the result of independent sample t-test indicated that no significant difference was found between gender groups (t_{diff} (18) = -0.578, p = .649).

As stated earlier, the tasks and test questions dealt with three types of spatial knowledge: Landmark, Route, and Survey. According to Fig. 12, the median scores of all three knowledge types improved from the pre- to post-test, and this shows that VRSVT had helped

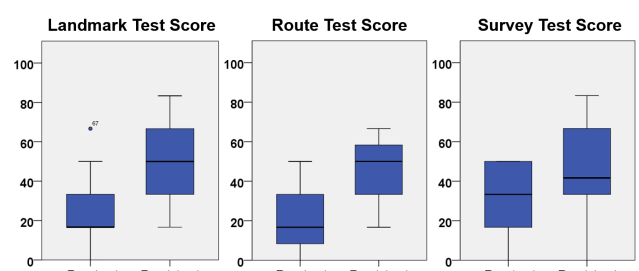

Figure 12: Boxplot of pre- and post-test results (in %) grouped based on Landmark (LEFT), Route (MIDDLE) and Survey (RIGHT) knowledge.

Table 2: Overall improvement in % of the three types of spatial knowledge.

	Mean Improvement	N	Std. deviation	Std. error mean
Landmark	25.00	20	23.258	5.201
Route	22.50	20	24.942	5.577
Survey	12.50	20	26.422	5.908

Table 3: Overall improvement in % of three types of spatial knowledge.

	Std. deviation	t-statistics	df	p-value
Landmark	23.258	-4.807	19	<.001
Route	24.942	-4.034	19	<.001
Survey	26.422	-2.116	19	=.048

participants gain a better understanding of all three knowledge types. Table 2 presents the descriptive data which indicates that Landmark knowledge saw the largest mean score improvement (from 25% to 50%), while Survey the least (from 33.33% to 45.83%).

Table 3 shows the results of one-sided paired-sample t-tests performed separately on the three different kinds of spatial knowledge. We found that there were significant improvements for all three of them. In particular, Landmark and Route knowledge experienced greater significant improvements (Landmark: t_{diff} (19) = -4.807, $p < .001$, Route: t_{diff} (19) = -4.034, $p < .001$) when compared to improvements for survey knowledge (Survey: t_{diff} (19) = -2.116, p =.048).

5.2 Subject Feedback

In this subsection, we present the findings from the subjective questionnaire given right after interacting with VRSVT. The main purpose of the questionnaire was to assess if participants engaged with VRSVT in meaningful ways (assessed through the VisEngage engagement survey devised by Hung and Parsons [20]) and their opinions and feelings towards the tool as a whole, and also towards each of the five individual visuo-interactive techniques (Solid Transition Maps, Synchronization, Dot-Manipulation, Rotation, and Geometric Magnification). We wanted to assess their usability and usefulness, and also explore design issues of VRSVT.

We organize the findings into three parts: (1) participants' affective response to the usefulness of the five individual techniques of VRSVT; (2) engagement questionnaire results; and (3) participants' comments and our observation.

5.2.1 Affective Response to Individual Techniques

Table 4 presents participants' overall response towards each of the visuo-interactive techniques. A 5-point Likert-scale questionnaire was used for participants to rate these individual techniques, where strongly positive responses were assigned a value of 5, and strongly negative responses a value of 1. The average response is 4.0 which indicates an overall positive response towards VRSVT as a whole. Individually, the mean scores for Rotation, Geometric Magnification, and Dot-Manipulation were quite high. There was a relatively neutral attitude towards STM and Synchronization.

5.2.2 Engagement Questionnaire Results

As mentioned earlier, we used the questionnaire reported in [20] to gauge participants' engagement level with VRSVT. Participants were asked to rate a set of 22 questions on a 7-point Likert-scale (7 for Strongly Agree, and 1 for Strongly Disagree). Every two answers would provide insight into each of the 11 different categories (Aesthetics, Captivation, Challenge, Control, Discovery, Exploration, Creativity, Attention, Interest, Novelty, and Autotelism).

Table 5 shows the descriptive data for the engagement scores. The mean engagement score for 20 participants is 132.8 out of 154 (SD = 12.27), which is 86.23% in percentage. page 7 shows the histogram

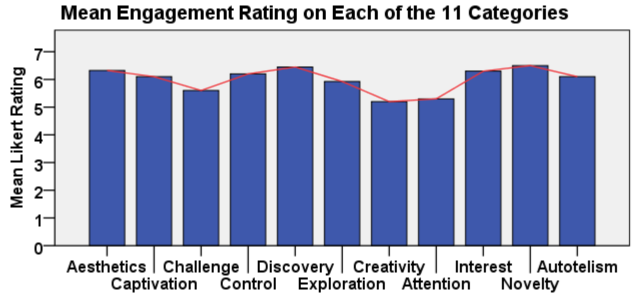

Figure 13: Histogram of the mean ratings of the visualization tool under each of the 11 categories of the user-visualization engagement survey.

of participants' mean rating of each of the 11 categories. As can be observed, the ratings for all categories were above 5 (Slightly Agree), which indicates that our participants were generally highly engaged with the tool.

We also carried out a reliability analysis on the VisEngage questionnaire and the Cronbach's alpha is shown to reach an acceptable reliability level (α= .850). This indicates the questionnaire is reliable in our study.

5.2.3 Participants' Comments and Our Observations

Based on our observations of participants' reactions during the experiment and their comments given during the post-experiment interviews, we found an overall positive reception toward VRSVT. Many participants were captivated by the VR visualization tool and commented that VR was *'intuitive'* and *'instructive'* to help them understand the geometric shapes and their transformational properties. When interacting with the geometric shapes, most of them started to engage quickly with the immersive world. We observed from the recorded videos of their body gestures that nearly all of them were quite engaged and focused on solving the tasks. After the experiment, some commented that *"It was really a great experience"* and *"It helped me learn a lot of interesting knowledge about geometric shapes and their transformations"*. However, there were some negative comments about the content being too difficult to follow (*"There are many things happening together sometimes, so I don't know where to look at"*). There were also some comments about tiredness and dizziness (*"when I use for a long time with virtual reality, I started to feel dizzy"*) commonly associated with using VR HMD for the first time or after a period of time.

6 Discussion, Limitations, and Future Work

In this section, we present the insights we gathered, discuss the limitations of this work, and propose some directions for future research.

Overall performance increment. From the statistical analysis, since the average score increased by 20% from pre- to post-test, we can infer that the VRSVT has had a positive effect on participants' explorations with the interactive visualizations. Liang and Sedig [34] showed that their interactive visualization tool displayed on 2D computer screens could be engaging for a variety of students (at a pre-university level). Our results would extend their findings and point to the viability of enhancing users' engagement with interactive visualizations in 3D VR environments. Compared to their work, which employed three consecutive days of training (45 minutes per day), our work shows that the overall performance can be significantly improved after only 40-50 minutes interacting in VRSVT. This finding is important because, as opposed to desktop 2D screens, a prolong use of VR can often be associated with motion sickness, tiredness, and dizziness. This would mean that

Table 4: Mean rating of the five visuo-interactive techniques.

	STM	Synchronization	Dot-Manipulation	Rotation	Geometric Magnification
Mean	3.60	3.45	4.45	4.50	3.95

Table 5: Overall descriptive data for engagement score.

	Mean	N	Std. deviation	Std. error mean
Engagement Score	132.750	20	12.273	2.744

short-focused periods of interaction is enough for users to acquire knowledge of the domain.

In addition, the participants in our study and [34] seemed to get similar performance improvement according to the test results. Although the settings are not totally the same, it probably suggests that participants can learn faster in VR environment than in the desktop PC environment. To make a fair comparison, we plan to conduct a larger experiment in the future to evaluate how different display types such as VR HMD, mobile tablets, and/or desktop PC could influence analytical reasoning activities and engagement levels.

Performance improvement with regard to landmark, route, survey knowledge. In terms of the three types of knowledge, the tool seems to have helped participants improve their scores significantly across the three knowledge types. Landmark knowledge and Route knowledge experienced greater improvement, which indicates that our VRSVT is especially helpful for supporting the learning of structural properties and transformation processes between the solids. However, in terms of Survey knowledge, although there was significant performance improvement (by 12.5%), it is considerably lower compared to Landmark (25.0%) and Route questions (22.5%). This was somewhat unexpected. One reason for this has to do with the affordances of VR HMD. In order to support the learning of Survey knowledge, we designed the STM and Synchronization techniques. However, the challenge we noticed was that the field of view of the VR HMD was limited and as such it was not easy for participants to be able to see multiple synchronized visualizations at the same time. The human visual system has a binocular field of view that exceeds 180 degrees horizontally, while current VR devices, such as the Oculus Rift we used in this research, are limited to around 90 degrees [62]. Future research is needed to explore how to design proper multiple visualizations that are linked together for current VR systems to support reasoning tasks. For example, one could explore different levels of scaling down these visualizations to a smaller size or use binocular presentation [65] to improve users' ability to see all the visualizations at once and follow concurrent changes.

Engagement level assessment. VRSVT obtained a high engagement score from participants as well, with a mean rating of 132.75 out of 154 (86.2%) in the VisEngage questionnaire. The categories of Interest, Discovery, and Novelty were rated high, which indicates that our participants considered VRSVT as a novel tool, and one which was easy to use and was supportive of discovery-based activities. However, the relatively low rating on the Attention category might provide an indication that participants were having difficulties paying attention to multiple visualizations changing simultaneously. From participants' comments and our direct observations during the experiment, the explorations in the virtual reality world gave them a feeling of novelty and brought forth active engagement. They tended to investigate and explore spontaneously, even with guided tasks. Additionally, as participants pointed out, the tiredness caused by wearing the VR HMD could lower their engagement and efficiency of the tool. It could be important to quantify the degree of simulator sickness by using, for example, Simulator Sickness Questionnaire proposed in [31] in the future.

Visuo-interactive techniques. According to the participants' affective response, Rotation and Dot-Manipulation techniques have received the highest ratings; we also observed participants used them very frequently. Geometric Magnification was also preferred by participants. STM and Synchronization were rated the lowest among the five techniques. According to this rating, we found these five techniques can be classified into two known categories: indirect and direct manipulation [50]. Indirect manipulation [48, 61] refers to interaction techniques where users use buttons, panels, toolbars or other interface elements to produce an effect on another element (or other elements); in our case, STM and Synchronization are indirect manipulations. Direct manipulations [2, 21, 56] refer to techniques which allow interacting with the visual elements directly, without the need of an intermediary interface element like the case of indirect interaction; the three direct manipulation techniques in our tool are Dot-Manipulation, Rotation, and Geometric Magnification. Our results suggest that participants tend to favor direct manipulation. Rotation and Geometric Magnification give participants the ability to directly manipulate the shapes and observe every single face of the solids. Likewise, Dot-Manipulation supported a direct and intuitive control of transformative processes. Positive comments that participants gave in the interviews show that these direct manipulation techniques, which allowed a more direct engagement with the solids, were favored and could facilitate exploration and learning processes with visualizations in virtual reality. The other aspect that explains the popularity of these direct manipulation techniques is because the VR controller and the technology favor this type of interaction because they attempt to replicate how things are manipulated in the physical environment. This finding suggests that it will be useful to incorporate direct manipulation techniques into educational visualization VR tools. On the other hand, indirect techniques have been shown to produce positive learning effects in the context of 2D displays (e.g. see [34, 50, 51, 53]). As such, further research is still needed to compare and evaluate what possible influences direct and indirect techniques could have on exploratory and learning processes with multiple visualization within VR systems.

Gender effect. While it has been shown that there are gender differences in problem solving in mathematics [66], our work shows no significant differences in performance between the two gender groups. Based on this finding, we hypothesize that VRSVT could support spatial reasoning activities independent of gender. However, more studies may still be needed to have a better understanding of how visualizations in VR could affect interaction for each gender group.

In all, our results provide empirical evidence that users can benefit from interacting with visualizations of non-trivial mathematical concepts in VR environments irrespective of gender and with only short interaction periods.

7 Summary and Conclusions

In this research, we have explored the level of analytical support and user engagement of interactive visualizations in virtual reality (VR) head-mounted display (HMD) environments. To do this exploration, we have designed and implemented VRSVT, a tool to help visualize 3D shapes for VR HMD with five different visuo-interactive techniques. We then have conducted an experiment with 20 university students (10 males, 10 females) to evaluate their knowledge of 3D geometry gained after interacting with the tool. We also have assessed their engagement level with the tool.

Our results show that interactive visualizations in VR can be supportive of exploration and sense-making activities that are conducive to knowledge acquisition. We have found that participants in our study are able to acquire knowledge in VR environments using a short training period, in contrast to similar studies that are performed with 2D displays, when longer sessions are needed. We also have found participants feel engaged and are able to perform meaningful exploratory and sense-making activities. They also find the interactions to be intuitive and the exploration novel. However, despite advances in VR HMD technology, a prolonged exploration with visualizations may still cause a certain degree of tiredness, which may lower the degree of engagement and efficiency. In addition, our results show that participants tend to favor more direct manipulations when immersed in the VR environment instead of indirect manipulations because of the need for participants to observe changes across multiple visualizations at the same time. What makes observing changes across visualizations difficult is the limited field of view of present VR HMD. Regardless of this challenge, the tool is found to be supportive of the needs of both male and female groups. While results of this research has shed some positive light on the use of visualizations in VR systems, further research is still needed if we are to design effective interactive visualizations for these systems.

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